

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

Cambridge Ordinary Level

## **MARK SCHEME for the October/November 2015 series**

### **4037 ADDITIONAL MATHEMATICS**

**4037/23**

Paper 2, maximum raw mark 80

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### Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

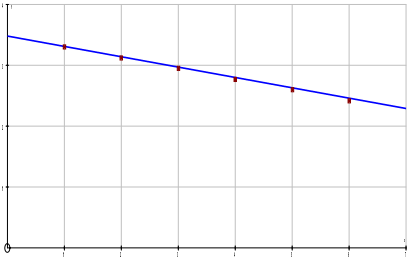
<b>1</b>	$y = x^3 + 3x^2 - 5x - 7$ $\frac{dy}{dx} = 3x^2 + 6x - 5$ $x = 2 \rightarrow \frac{dy}{dx} = 19$ $y = 3$ $\text{eqn of tangent: } \frac{y-3}{x-2} = 19 \rightarrow (y = 19x - 35)$	M1 A1 A1FT B1 A1FT	Differentiate  on <i>their</i> $\frac{dy}{dx}$
<b>2</b>	$2x + k + 2 = 2x^2 + (k + 2)x + 8$ $2x^2 + kx + 6 - k = 0$ $b^2 - 4ac = k^2 - 4 \times 2(6 - k)$ $k^2 + 8k - 48 > 0$ $(k + 12)(k - 4) > 0$ $k < -12 \text{ or } k > 4$	M1 A1 M1  DM1 A1 A1	eliminate $y$ or $x$ correct quadratic use discriminant  attempt to solve 3 term quadratic $k = -12$ and $k = 4$
<b>3 (a)</b>	$\frac{dy}{dx} = \frac{(2 - x^2)3x^2 - x^3(-2x)}{(2 - x^2)^2} = \left( \frac{6x^2 - x^4}{(2 - x^2)^2} \right)$	M1 A2,1,0	For quotient rule (or product rule on correct $y$ )
<b>(b)</b>	$\frac{dy}{dx} = x \times \frac{1}{2}(4x + 6)^{-0.5} \times 4 + (4x + 6)^{0.5}$ $= \frac{6(x+1)}{(4x+6)^{0.5}} \rightarrow k = 6$	M1 A1 A1	product rule
<b>4</b>	$x(4 - \sqrt{3}) = 13$ $x = \frac{13(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$ $= 4 + \sqrt{3}$ $y = 1 - 2\sqrt{3}$	M1 A1 M1 A1 A1	eliminate $y$ or $x$ simplified rationalisation

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5	$(x-3)(x-3)(x-1) = 0$ $x^3 - 7x^2 + 15x - 9 = 0$ $a = -7$ $b = 15$ $c = -9$	M1  A1 A1 A1	AG for c
6	$\log_x 2 = \frac{\log_2 2}{\log_2 x}$ $2 \log_2 x = \log_2 x^2$ $3 = \log_2 8$ $8x^2 - 29x + 15 (= 0)$ $\rightarrow (8x-5)(x-3) (= 0)$ $x = \frac{5}{8}$ or $x = 3$	B1  B1 B1 M1  A1	obtain quadratic and attempt to solve
7 (i)	$a = -\frac{20}{(t+2)^3}$  $t = 3 \rightarrow a = -0.16 \text{ m/s}^2$	M1 A1  A1FT	$k(t+2)^{-3}$ oe $k = -20$
(ii)	$\frac{10}{(t+2)^2}$ is never zero.	B1	
(iii)	$s = -\frac{10}{t+2} + 5$	M1 A1 A1	integrate $\frac{k}{t+2}$ $k = -10$ +5
(iv)	$s = \left[ -\frac{10}{t+2} \right]_3^8 = -1 + 2$ = 1	M1  A1	insert limits and subtract

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8	(i)	$\sec^2 x + \operatorname{cosec}^2 x = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$ $= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$ $= \frac{1}{\sin^2 x \cos^2 x}$ $= \sec^2 x \operatorname{cosec}^2 x$	B1 B1 B1 B1	add fractions use of $\sin^2 x + \cos^2 x = 1$ fully correct solution
	(ii)	$\frac{1}{\cos^2 x \sin^2 x} = 4 \frac{\sin^2 x}{\cos^2 x}$ $\rightarrow 4 \sin^2 x = 1$ $\sin x = \pm \frac{1}{\sqrt{2}}$ $x = 135^\circ, 225^\circ$	M1 A1 A1, A1	correct simplified equation
9	(i)	$f(x) = 3x^2 + 12x + 2 = 3(x+2)^2 - 10$ $a = 3$ $b = 2$ $c = -10$	B1 B1 B1	
	(ii)	<p>minimum <math>f(x) = -10</math> at <math>x = -2</math></p>	B1FT B1FT	
	(iii)	$f\left(\frac{1}{y}\right) = 0 \rightarrow \left(\frac{1}{y}\right) = (\pm)\sqrt{\frac{10}{3}} - 2$ $y = -5.74, -0.26$	M1 A1, A1	obtain explicit expression for $\frac{1}{y}$ or $y$

10 (i)	$\frac{d}{dx}(e^{2-x^2}) = -2xe^{2-x^2}$	B1	$k = -2$																					
(ii)	$-\frac{3e^{2-x^2}}{2} + c$	M1 A1FT	$De^{2-x^2}$ $D = \frac{-3}{2}$ or $\frac{3}{k}$																					
(iii)	$\left[ -\frac{3e^{2-x^2}}{2} \right]_1^{\sqrt{2}} = -\frac{3}{2} + \frac{3}{2}e$ 2.58	M1 A1	insert limits on <i>their</i> (ii) and subtract																					
(iv)	$y = 3xe^{2-x^2}$ $\frac{dy}{dx} = 3x(-2xe^{2-x^2}) + 3e^{2-x^2}$ $\frac{dy}{dx} = 0 \rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm 0.707$ $y = \pm \frac{3}{\sqrt{2}}e^{1.5} = \pm 9.51$	M1 A1  A1 A1	product rule  both $x$ or a pair  both $y$																					
11 (i)	$\log N = \log A - t \log b$	B1																						
(ii)	<table border="1"> <tr> <td><math>t</math></td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> </tr> <tr> <td><math>\log N</math></td> <td>3.30</td> <td>3.11</td> <td>2.95</td> <td>2.77</td> <td>2.60</td> <td>2.41</td> </tr> <tr> <td><math>\ln N</math></td> <td>7.60</td> <td>7.17</td> <td>6.79</td> <td>6.38</td> <td>5.98</td> <td>5.56</td> </tr> </table> 	$t$	1	2	3	4	5	6	$\log N$	3.30	3.11	2.95	2.77	2.60	2.41	$\ln N$	7.60	7.17	6.79	6.38	5.98	5.56	M1	find logs of $N$
$t$	1	2	3	4	5	6																		
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$\ln N$	7.60	7.17	6.79	6.38	5.98	5.56																		
(iii)	gradient = $-\log b = \frac{2.415 - 3.3}{5} \rightarrow b = 1.5$ intercept = $\log A = 3.47 \rightarrow A = 2950$	DM1 DM1 A1	set gradient = $-\log b$ and solve set intercept = $\log A$ and solve both values correct																					
(iv)	$t = 10 \rightarrow N = \frac{2950}{1.5^{10}} = 51$	B1																						
(v)	$N = 10 \rightarrow 1.5^t = 295 \rightarrow t = \frac{\log 295}{\log 1.5}$ $= 14$ years	M1 A1	substitute $N = 10$ , <i>their</i> $A$ , $b$ into given or transformed equation																					

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12	$v_p = \begin{pmatrix} 250 \cos 20^\circ \\ 250 \sin 20^\circ \end{pmatrix}, v_r = \begin{pmatrix} V \cos 30^\circ \\ V \sin 30^\circ \end{pmatrix}, v_w = \begin{pmatrix} 0 \\ w \end{pmatrix}$	B1	
	$v_r = v_p + v_w$ $\begin{pmatrix} V \cos 30^\circ \\ V \sin 30^\circ \end{pmatrix} = \begin{pmatrix} 250 \cos 20^\circ \\ 250 \sin 20^\circ \end{pmatrix} + \begin{pmatrix} 0 \\ w \end{pmatrix}$		
	$V = \frac{250 \cos 20^\circ}{\cos 30^\circ}$ $= 271 \text{ km/hr}$	M1 A1	equate x components and solve
	$w = V \sin 30^\circ - 250 \sin 20^\circ$ $= 50.1 \text{ km/hr}$	M1 A1	equate y components and solve
	<b>OR</b> triangle with sides 250 $V$ $w$ opposite angles 60°    110°    10°	B1	
	sine rule: $\frac{w}{\sin 10^\circ} = \frac{250}{\sin 60^\circ}$ $w = 50.1 \text{ km/hr}$ $\frac{V}{\sin 110^\circ} = \frac{250}{\sin 60^\circ}$ $V = 271 \text{ km/hr}$	M1 A1 M1 A1	apply to correct triangle and solve apply to correct triangle and solve